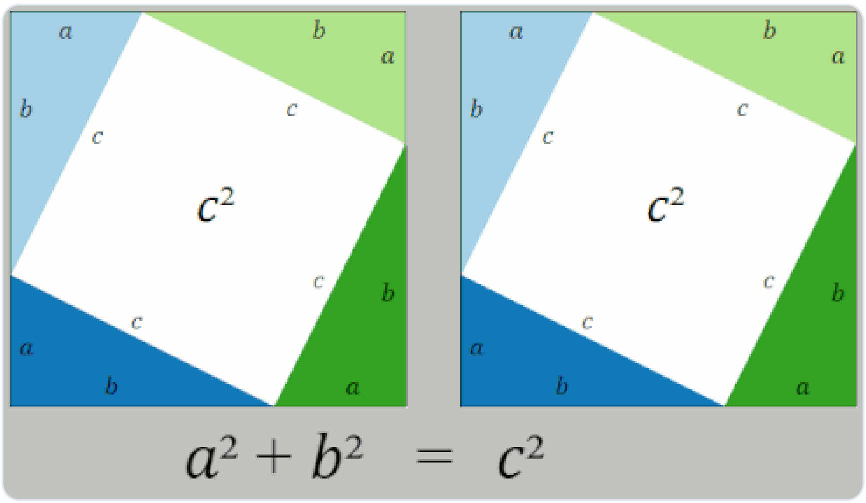
Pythagorean Theorem · Lecture 02 · `geo-pythagorean`

📺 **Course materials — Lecture 02 (94 min):** lecture video · handout PDF (Lecture 2) · board notes (02.jpg) · homework answer key (Chapter 2) — all in the `amc8/video` folder. · 🖨️ **Printable PDF** (answers & solutions at the back)

🔑 Key facts

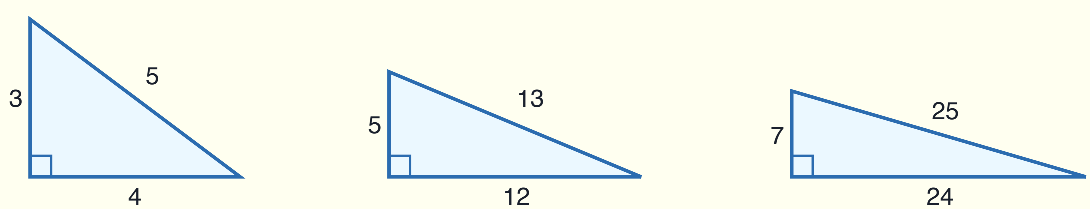
In a right triangle with legs a, b and hypotenuse c: $a^2 + b^2 = c^2$. The **converse** is just as useful on AMC 8: if $a^2 + b^2 = c^2$, the angle opposite c is a right angle — see Example 2, where spotting 5-12-13 *reveals* a hidden right angle.



Why it works — the picture proof: the same four copies of the triangle sit inside the same big square two ways; what's left over is c^2 on one side and $a^2 + b^2$ on the other. (Animated version on the wiki page.)

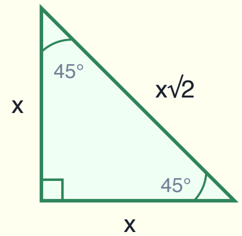
🧰 Instructor's toolbox (from the lecture video & board notes)

1 · Memorize the triples. Nearly every AMC 8 Pythagorean problem uses one of these primitive triples: 3-4-5, 5-12-13, 8-15-17, 7-24-25, 9-40-41 — or a multiple (6-8-10, 9-12-15, 10-24-$26\ldots$). Seeing 12 and 13 together should make you think "5!" before you compute anything.

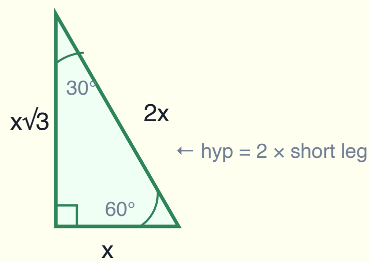


2 · Un-scale to find hidden triples. Big numbers usually hide a small triple: divide by the common factor first. The instructor's example: 18-24-30 — divide by 6 and it's just 3-4-5. In Example 3 below, $\sqrt{25^2 - 24^2}$ is instant if you recognize 7-24-25; no squaring needed.

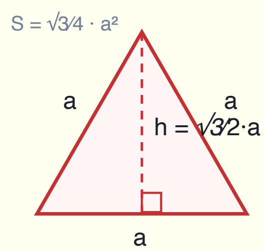
3 · 45-45-90 (isosceles right). Sides $x : x : x\sqrt{2}$. **Shortcut:** given only the hypotenuse c, its area is $\dfrac{c^2}{4}$ — the instructor's one-liner (hypotenuse 10 → area $10^2/4 = 25$). This is exactly how the mountain problem (Practice 17) becomes "overlap $= h^2$".



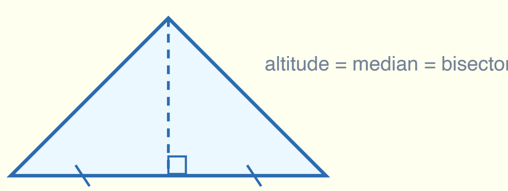
4 · 30-60-90. Sides $x : x\sqrt{3} : 2x$. **Careful:** the hypotenuse is twice the side *opposite the* $30°$ *angle* (the short leg) — doubling the wrong leg is the classic error the instructor calls out. Recognizing "$CB = \frac{1}{2}CM$" is what cracks Challenge C1.



5 · Equilateral triangle formulas — "on 99.9% of exams" (instructor's words). Side a: height $h = \dfrac{\sqrt{3}}{2}a$, area $S = \dfrac{\sqrt{3}}{4}a^2$. Both come from cutting the triangle into two 30-60-90s — used in Practice 12 and 14.

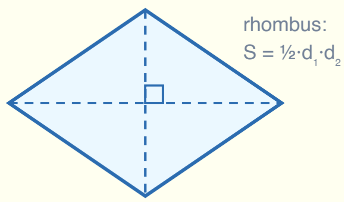


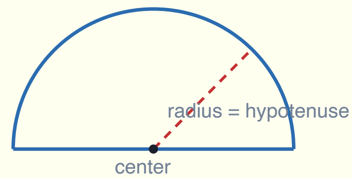
6 · Isosceles "three-in-one" line. In an isosceles triangle, the altitude, median, and apex-angle bisector to the base are *the same line* — so the base is automatically bisected (Example 4: $AH = 21$ with no work).



7 · Make your own right triangles. The theorem is useless without a right triangle, so build one:

- **Trapezoid** → drop heights from both ends of the short base (2 right triangles + a rectangle) — Example 3;
- **Isosceles triangle** → the altitude to the base bisects it — Example 4;
- **Circle / semicircle** → connect the center to a corner or chord endpoint; a radius becomes a hypotenuse — Practice 2, 3, 4;
- **Rhombus** → diagonals are perpendicular and bisect each other — Example 1. Bonus: $S_{\text{rhombus}} = \frac{1}{2}d_1 d_2$ (a rhombus is just a parallelogram with four equal sides);
- **$45°$ or $30°/60°$ angles anywhere** → drop an altitude to trap them in right triangles — Practice 8.





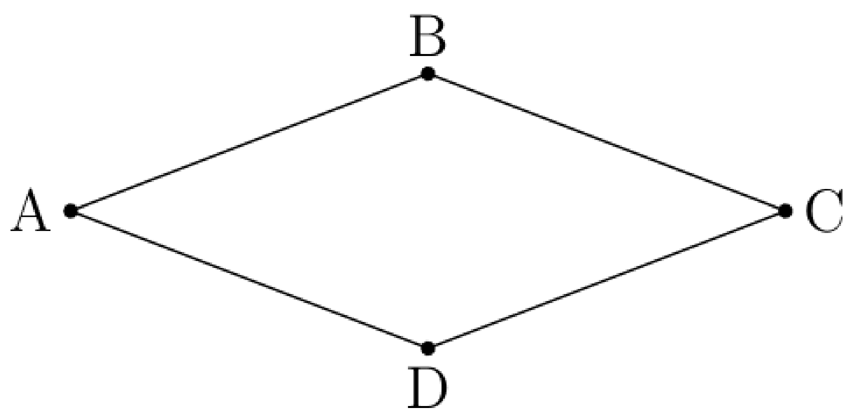
8 · Radical survival kit. Pythagorean answers rarely come out whole, so simplify like the instructor: $\sqrt{A \cdot B} = \sqrt{A} \cdot \sqrt{B}$ and $\sqrt{\frac{A}{B}} = \frac{\sqrt{A}}{\sqrt{B}}$. To simplify, factor out the *perfect square*: $\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$ (choose 4×5, not 2×10); $\sqrt{32} = 4\sqrt{2}$; $\sqrt{200} = 10\sqrt{2}$. To clear a root from a denominator: $\sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$. Know your perfect squares to $20^2 = 400$.

9 · Angle-sum quickies (used with the theorem constantly). Interior angles of an n-gon sum to $(n-2) \cdot 180°$; exterior angles of *any* polygon sum to $360°$ (regular: $120° \times 3$, $90° \times 4$, $72° \times 5$, $60° \times 6$). That's how Practice 11 finds the $90°$ hiding in the hexagon. Also: three sides determine a triangle completely (SSS), three angles only determine its *shape* — and side-side-angle is **not** a congruence test.

📖 Worked examples

2019 AMC 8 #4 · rhombus

Example 1. Quadrilateral $ABCD$ is a rhombus with perimeter 52 meters. The length of diagonal $\overline{AC}$ is 24 meters. What is the area in square meters of rhombus $ABCD$?



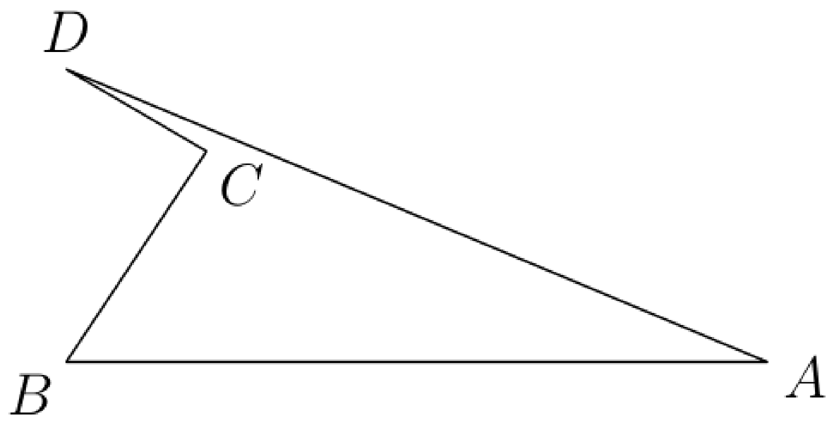
(A) 60 **(B)** 90 **(C)** 105 **(D)** 120 **(E)** 144

🧰 #7 Build a right triangle #1 Triples

Key idea: a rhombus's diagonals are perpendicular and bisect each other. Where they cross at O: each side is $52/4 = 13$ and $AO = 24/2 = 12$ — that's the 5-12-13 triple, so $BO = 5$. Then $S_{\triangle ABO} = \frac{1}{2} \cdot 5 \cdot 12 = 30$ and the rhombus is 4 such triangles: $4 \cdot 30 = 120$. **Answer: D**
Shortcut: $S = \frac{1}{2}d_1 d_2 = \frac{1}{2} \cdot 24 \cdot 10 = 120$.

2017 AMC 8 #18 · converse

Example 2. In the non-convex quadrilateral $ABCD$ shown below, $\angle BCD$ is a right angle, $AB = 12$, $BC = 4$, $CD = 3$, and $AD = 13$. What is the area of quadrilateral $ABCD$?



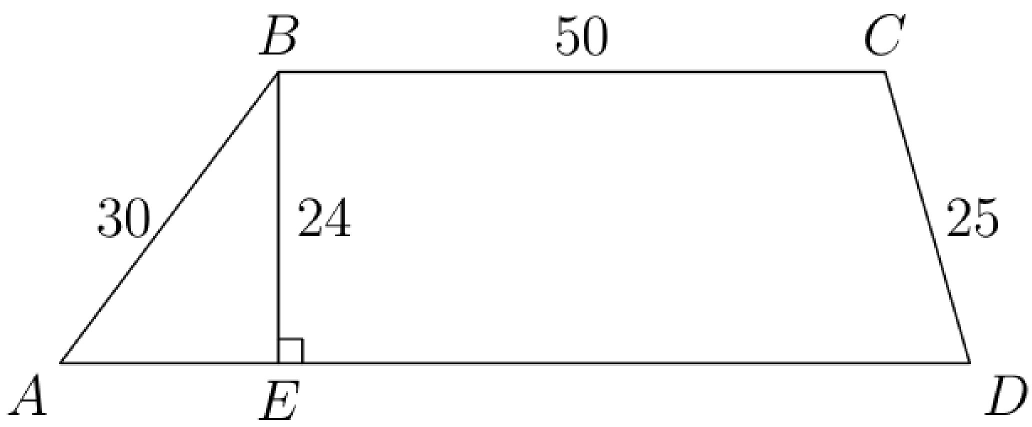
(A) 12 **(B)** 24 **(C)** 26 **(D)** 30 **(E)** 36

🧰 #1 Triples Converse

Key idea: draw $\overline{BD}$. In right $\triangle BCD$: 3-4-5, so $BD = 5$. Now look at $\triangle ABD$: sides $5, 12, 13$ — the converse says $\angle ABD = 90^\circ$! So the area is the big right triangle minus the small one: $\frac{1}{2} \cdot 12 \cdot 5 - \frac{1}{2} \cdot 4 \cdot 3 = 30 - 6 = 24$. **Answer: B**

2005 AMC 8 #19 · trapezoid

Example 3. What is the perimeter of trapezoid $ABCD$?



(A) 180 **(B)** 188 **(C)** 196 **(D)** 200 **(E)** 204

🧰 #7 Build a right triangle #2 Un-scale

Key idea: the classic trapezoid move — drop heights from both ends of the top base, splitting the figure into two right triangles and a rectangle. Left triangle: $AE = \sqrt{30^2 - 24^2} = 18$. Right triangle: $DF = \sqrt{25^2 - 24^2} = 7$. So $AD = 18 + 50 + 7 = 75$ and $P = 50 + 30 + 75 + 25 = 180$. **Answer: A**

2015 AMC 8 #6 · isosceles

Example 4. In $\triangle ABC$, $AB = BC = 29$, and $AC = 42$. What is the area of $\triangle ABC$?

(A) 100 **(B)** 420 **(C)** 500 **(D)** 609 **(E)** 701

🧰 #6 Three-in-one #1 Triples

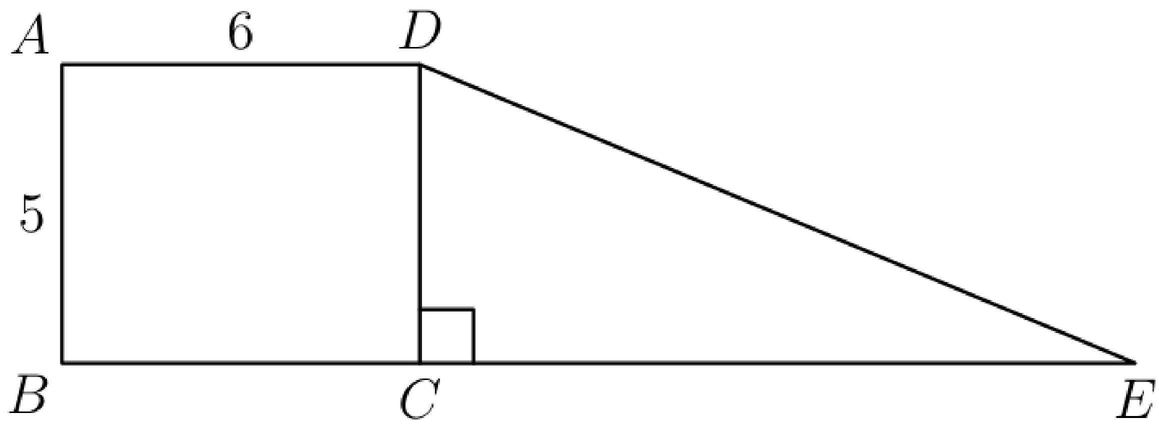
Key idea: in an isosceles triangle the altitude to the base bisects it. Drop $BH \perp AC$: then $AH = 21$ and $BH = \sqrt{29^2 - 21^2} = \sqrt{400} = 20$ (a 20-21-29 triple!). Area $= \frac{1}{2} \cdot 42 \cdot 20 = 420$. **Answer: B**

✏️ Practice

Try each one before opening the solution. Figures are from the official exams.

2014 AMC 8 #14

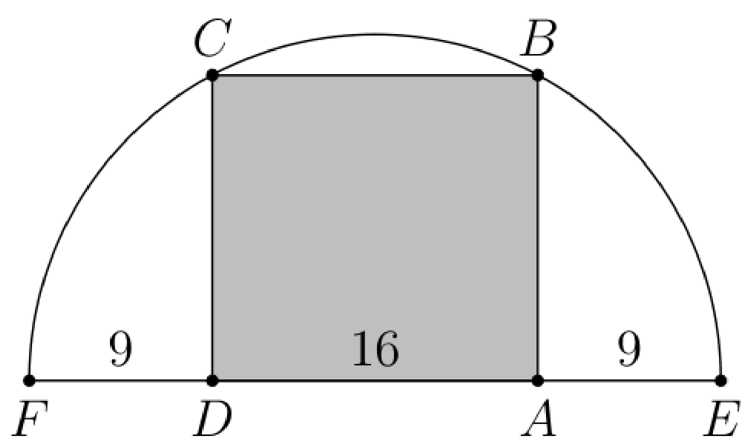
1. Rectangle $ABCD$ and right triangle DCE have the same area. They are joined to form a trapezoid, as shown. What is DE?



(A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

2020 AMC 8 #18

2. Rectangle $ABCD$ is inscribed in a semicircle with diameter $\overline{FE}$, as shown. Let $DA = 16$, and $FD = AE = 9$. What is the area of $ABCD$?



(A) 240 **(B)** 248 **(C)** 256 **(D)** 264 **(E)** 272

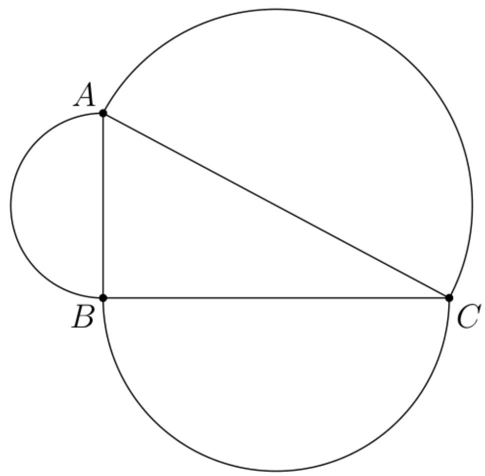
2013 AMC 8 #20

3. A 1×2 rectangle is inscribed in a semicircle with the longer side on the diameter. What is the area of the semicircle?

(A) $\frac{\pi}{2}$ **(B)** $\frac{2\pi}{3}$ **(C)** π **(D)** $\frac{4\pi}{3}$ **(E)** $\frac{5\pi}{3}$

2013 AMC 8 #23

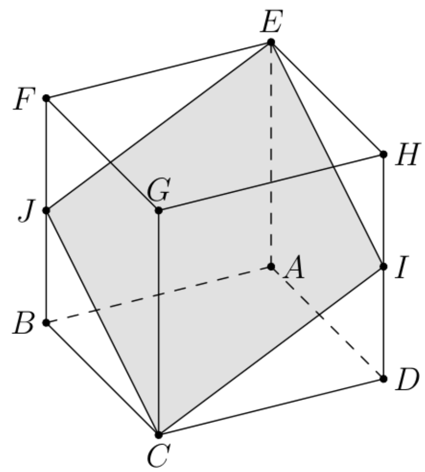
4. $\angle ABC$ of $\triangle ABC$ is a right angle. The sides of $\triangle ABC$ are the diameters of semicircles as shown. The area of the semicircle on $\overline{AB}$ equals 8π, and the arc of the semicircle on $\overline{AC}$ has length 8.5π. What is the radius of the semicircle on $\overline{BC}$?



(A) 7 **(B)** 7.5 **(C)** 8 **(D)** 8.5 **(E)** 9

2018 AMC 8 #24

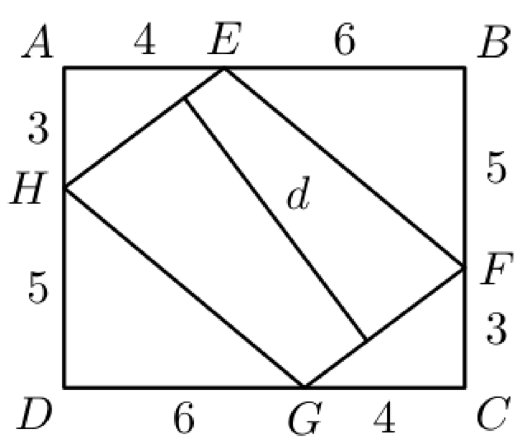
5. In the cube $ABCDEFGH$ with opposite vertices C and E, J and I are the midpoints of segments $\overline{FB}$ and $\overline{HD}$, respectively. Let R be the ratio of the area of the cross-section $EJCI$ to the area of one of the faces of the cube. What is R^2?



(A) $\frac{5}{4}$ **(B)** $\frac{4}{3}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{25}{16}$ **(E)** $\frac{9}{4}$

2004 AMC 8 #24

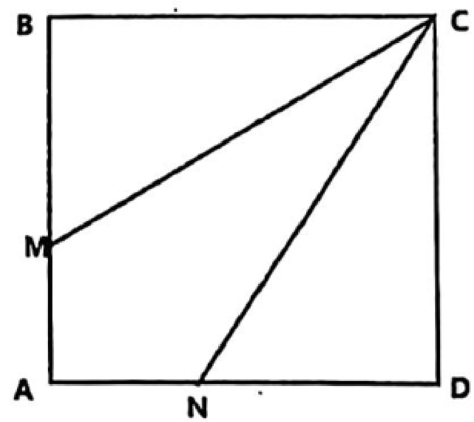
6. In the figure, $ABCD$ is a rectangle and $EFGH$ is a parallelogram. Using the measurements given in the figure, what is the length d of the segment that is perpendicular to $\overline{HE}$ and $\overline{FG}$?



(A) 6.8 **(B)** 7.1 **(C)** 7.6 **(D)** 7.8 **(E)** 8.1

mock · course problem

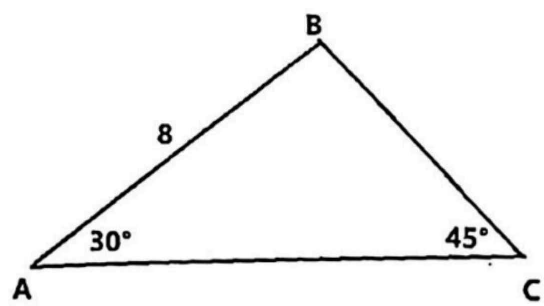
7. Square $ABCD$ has sides of length 3. Segments $\overline{CM}$ and $\overline{CN}$ divide the square's area into three equal parts. How long is the segment connecting M and N?



mock · course problem

8. Triangle ABC is shown with $AB = 8$, $\angle A = 30^\circ$, $\angle C = 45^\circ$. Find BC.

(A) 4　　(B) $4\sqrt{2}$　　(C) $4\sqrt{3}$

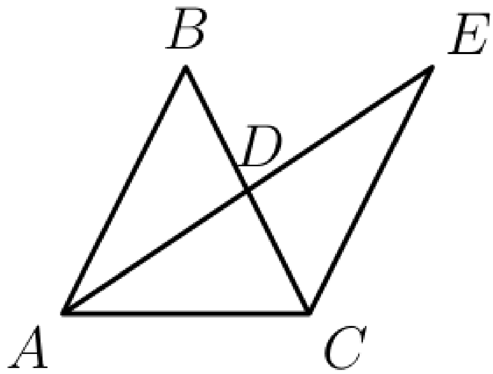


关键思路解析:

B

2006 AMC 8 #19

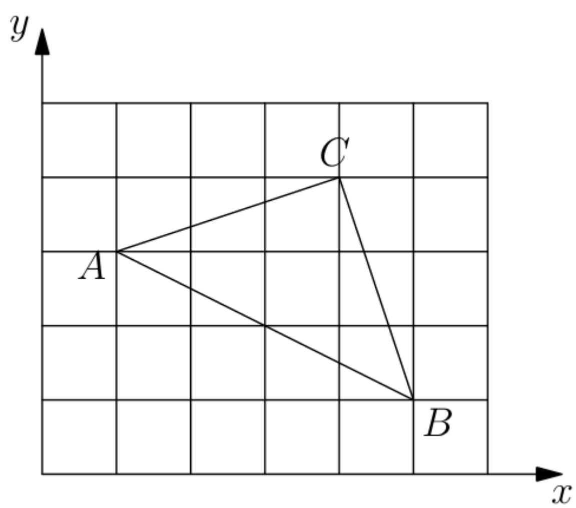
9. Triangle ABC is an isosceles triangle with $\overline{AB} = \overline{BC}$. Point D is the midpoint of both $\overline{BC}$ and $\overline{AE}$, and $\overline{CE}$ is 11 units long. Triangle ABD is congruent to triangle ECD. What is the length of $\overline{BD}$?



(A) 4 **(B)** 4.5 **(C)** 5 **(D)** 5.5 **(E)** 6

2015 AMC 8 #19

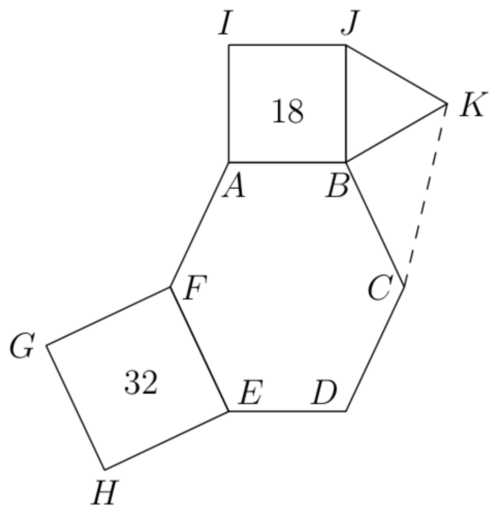
10. A triangle with vertices $A = (1, 3)$, $B = (5, 1)$, $C = (4, 4)$ is plotted on a 6×5 grid. What fraction of the grid is covered by the triangle?



(A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

2015 AMC 8 #21

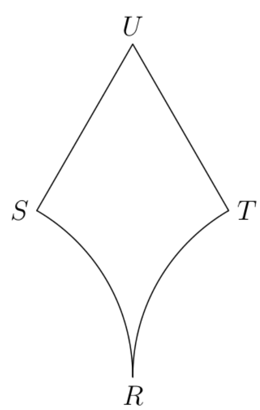
11. In the figure, hexagon $ABCDEF$ is equiangular, $ABJI$ and $FEHG$ are squares with areas 18 and 32 respectively, $\triangle JBK$ is equilateral and $FE = BC$. What is the area of $\triangle KBC$?



(A) $6\sqrt{2}$ **(B)** 9 **(C)** 12 **(D)** $9\sqrt{2}$ **(E)** 32

2017 AMC 8 #25

12. In the figure shown, $\overline{US}$ and $\overline{UT}$ are line segments each of length 2, and $m\angle TUS = 60^\circ$. Arcs $\overset{\frown}{TR}$ and $\overset{\frown}{SR}$ are each one-sixth of a circle with radius 2. What is the area of the region shown?

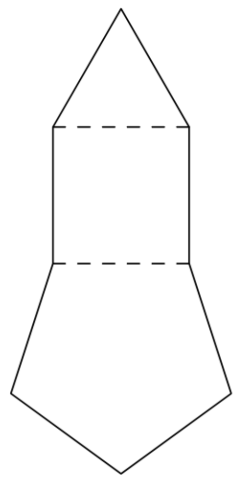


A) $3\sqrt{3} - \pi$ **B)** $4\sqrt{3} - \frac{4\pi}{3}$ **C)** $2\sqrt{3}$ **D)** $4\sqrt{3} - \frac{2\pi}{3}$ **E)** $4 + \frac{4\pi}{3}$

(A) $3\sqrt{3} - \pi$ **(B)** $4\sqrt{3} - \frac{4\pi}{3}$ **(C)** $2\sqrt{3}$ **(D)** $4\sqrt{3} - \frac{2\pi}{3}$ **(E)** $4 + \frac{4\pi}{3}$

2009 AMC 8 #9

13. Construct a square on one side of an equilateral triangle. On one non-adjacent side of the square, construct a regular pentagon, as shown. On a non-adjacent side of the pentagon, construct a hexagon. Continue until you construct an octagon. How many sides does the resulting polygon have?



(A) 21 **(B)** 23 **(C)** 25 **(D)** 27 **(E)** 29

2012 AMC 8 #23

14. An equilateral triangle and a regular hexagon have equal perimeters. If the triangle's area is 4, what is the area of the hexagon?

(A) 4 **(B)** 5 **(C)** 6 **(D)** $4\sqrt{3}$ **(E)** $6\sqrt{3}$

🆕 Fresh from the latest exams

The same ideas, exactly as they appeared on the most recent tests.

2026 AMC 8 #3

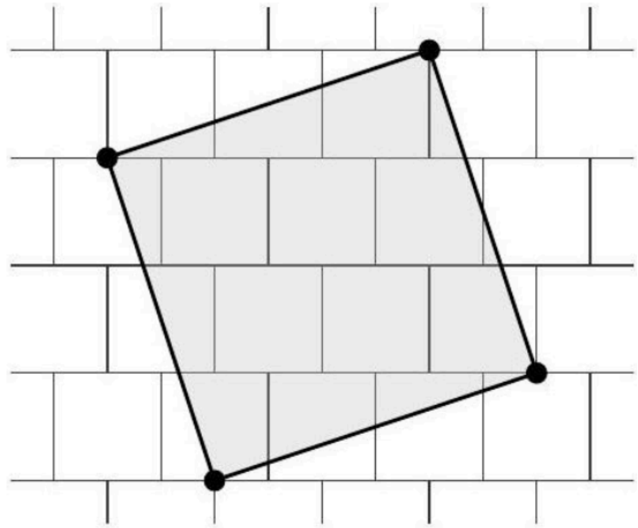
15. Haruki has a piece of wire that is 24 centimeters long. He wants to bend it to form each of the following shapes, one at a time: (1) a regular hexagon with side length 5 cm; (2) a square of area 36 cm²; (3) a right triangle whose legs are 6 and 8 cm long. Which of the shapes can Haruki make?

(A) Triangle only **(B)** Hexagon and square only **(C)** Hexagon and triangle only

(D) Square and triangle only **(E)** Hexagon, triangle, and square

2026 AMC 8 #13

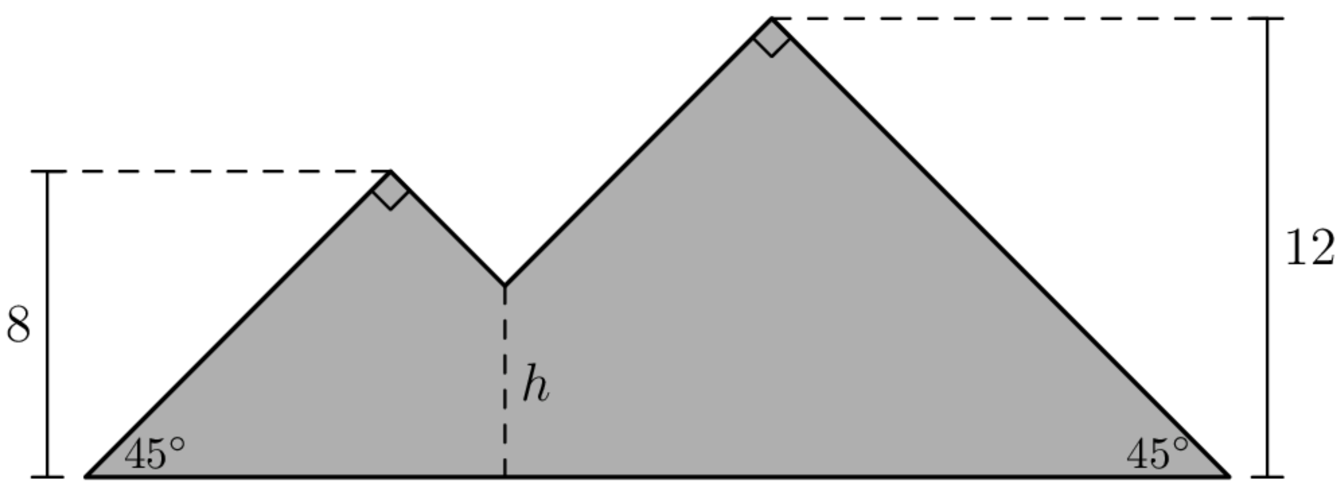
16. The figure below shows a tiling of 1×1 unit squares. Each row of unit squares is shifted horizontally by half a unit relative to the row above it. A shaded square is drawn on top of the tiling. Each vertex of the shaded square is a vertex of one of the unit squares. In square units, what is the area of the shaded square?



(A) 10 **(B)** $\frac{21}{2}$ **(C)** $\frac{32}{3}$ **(D)** 11 **(E)** $\frac{34}{3}$

2024 AMC 8 #24

17. Jean has made a piece of stained glass art in the shape of two mountains, as shown in the figure below. One mountain peak is 8 feet high while the other peak is 12 feet high. Each peak forms a 90° angle, and the straight sides form a 45° angle with the ground. The artwork has an area of 183 square feet. The sides of the mountain meet at an intersection point near the center of the artwork, h feet above the ground. What is the value of h?



(A) 4 **(B)** 5 **(C)** $4\sqrt{2}$ **(D)** 6 **(E)** $5\sqrt{2}$

⭐ Challenge (AMC 10 level)

2011 AMC 10B #18 · 30-60-90

C1. Rectangle $ABCD$ has $AB = 6$ and $BC = 3$. Point M is chosen on side $\overline{AB}$ so that $\angle AMD = \angle CMD$. What is the degree measure of $\angle AMD$?

(A) 15 **(B)** 30 **(C)** 45 **(D)** 60 **(E)** 75

2015 AMC 10A #19 · 30-60-90

C2. The isosceles right triangle ABC has right angle at C and area 12.5. The rays trisecting $\angle ACB$ intersect AB at D and E. What is the area of $\triangle CDE$?

(A) $\frac{5\sqrt{2}}{3}$ **(B)** $\frac{50\sqrt{3}-75}{4}$ **(C)** $\frac{15\sqrt{3}}{8}$ **(D)** $\frac{50-25\sqrt{3}}{2}$ **(E)** $\frac{25}{6}$

Sources: prep-course Lecture 02 (video + board notes + 20 worked examples, translated to English) and official AMC 8 problems from the local bank (`kb/content/banks/`). Answers verified against official AoPS answer keys where available. Proof animations from Wikimedia Commons (CC BY-SA 4.0): rearrangement proof by William B. Faulk; shearing proof by RajRaizada.

🗝️ Answer key

Example	1	2	3	4													
	D	B	A	B													
Practice	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
	B	A	C	B	C	C	$\sqrt{2}$	$4\sqrt{2}$	D	A	C	B	B	C	D	A	B
Challenge	C1	C2															
	E	D															

📝 Solutions to practice problems

Practice 1.

🧰 #1 Triples

$S_{\triangle DCE} = S_{ABCD} = 30$, so $\frac{1}{2} \cdot CE \cdot 5 = 30 \Rightarrow CE = 12$. In right $\triangle DCE$ with legs 5 and 12: $DE = 13$. **Answer: B**

Practice 2.

🧰 #7 Build a right triangle #1 Triples

Let O be the center. The radius is $OB = (9 + 16 + 9)/2 = 17$ and $OA = 8$. In right $\triangle AOB$: $AB = \sqrt{17^2 - 8^2} = 15$ (the 8-15-17 triple). Area $= 15 \cdot 16 = 240$. **Answer: A**

Practice 3.

🧰 #7 Build a right triangle #8 Radicals

Connect the center O to a top corner B: $OB = \sqrt{1^2 + 1^2} = \sqrt{2}$ — that's the radius. Semicircle area $= \frac{1}{2}\pi(\sqrt{2})^2 = \pi$. **Answer: C**

Practice 4.

🧰 #7 Build a right triangle #1 Triples

Semicircle on AB: $\frac{1}{2}\pi(AB/2)^2 = 8\pi \Rightarrow AB = 8$. Arc on AC: $\frac{1}{2}\pi\, AC = 8.5\pi \Rightarrow AC = 17$. Then $BC = \sqrt{17^2 - 8^2} = 15$ (8-15-17 again), so the radius is 7.5. **Answer: B**

Practice 5.

#7 Build a right triangle · #8 Radicals

By SAS, $\triangle EFJ \cong \triangle CBJ \cong \triangle CDI \cong \triangle HEI$, so $EJCI$ is a rhombus — use $S = \frac{1}{2}d_1 d_2$. With side 1: diagonal $JI = BD = \sqrt{2}$; space diagonal $CE = \sqrt{1^2+1^2+1^2} = \sqrt{3}$. $S = \frac{1}{2}\sqrt{2}\sqrt{3} = \frac{\sqrt{6}}{2}$, so $R = \frac{\sqrt{6}}{2}$ and $R^2 = \frac{3}{2}$. **Answer: C**

Practice 6.

#1 Triples · #7 Build a right triangle

Area two ways. $HE = \sqrt{3^2+4^2} = 5$. Parallelogram area $=$ rectangle $-$ four corner right triangles $= 80-6-15-6-15 = 38$. But the area is also $HE \cdot d = 5d$, so $d = 38/5 = 7.6$. **Answer: C**

Practice 7.

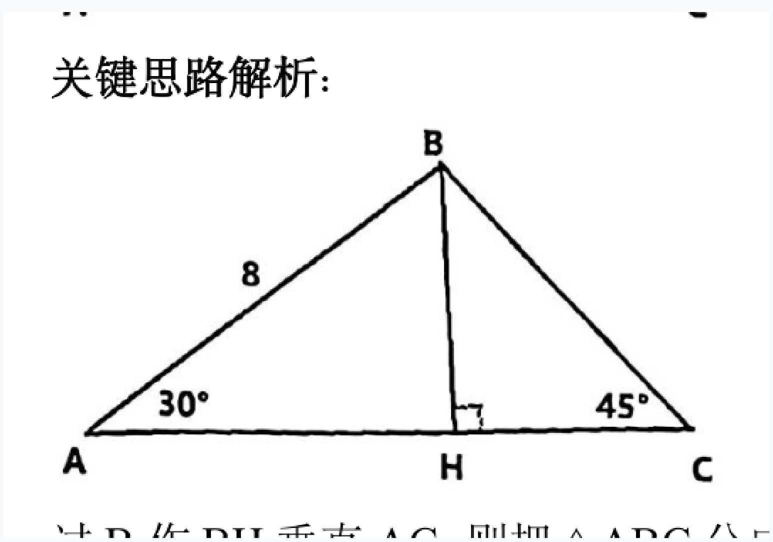
#3 45-45-90

Each part has area 3: $\frac{1}{2} \cdot BM \cdot 3 = 3 \Rightarrow BM = 2$, so $AM = 1$; likewise $AN = 1$. $\triangle AMN$ is an isosceles right triangle: $MN = \sqrt{2}$. **Answer:** $\sqrt{2}$

Practice 8.

#7 Build a right triangle · #4 30-60-90 · #3 45-45-90

Drop $BH \perp AC$ to split into two special triangles. In the 30-60-90 triangle ABH: $BH = 8/2 = 4$. In the 45-45-90 triangle BHC: $BC = 4\sqrt{2}$.



Answer: $4\sqrt{2}$ **(B)**

Practice 9.

#9 Angles & congruence

From the congruence, $AB = CE = 11$; isosceles gives $BC = AB = 11$, so $BD = 11/2 = 5.5$.

Answer: D

Practice 10.

#9 Angles & congruence | Converse

Congruent right triangles (1-3 legs) show $AC = CB = \sqrt{10}$ and $\angle ACB = 90^\circ$, so the area is $\frac{1}{2}\sqrt{10} \cdot \sqrt{10} = 5$. The grid is 30, so the fraction is $\frac{1}{6}$. (Or: box the triangle and subtract the three corner triangles.)

Answer: A

Practice 11.

#9 Angles & congruence | #8 Radicals

Equiangular hexagon → each interior angle 120°. Around B: $\angle ABC = 120^\circ$, $\angle JBA = 90^\circ$, $\angle JBK = 60^\circ$, so $\angle KBC = 90^\circ$ — a right triangle! $KB = JB = \sqrt{18} = 3\sqrt{2}$ and $BC = FE = \sqrt{32} = 4\sqrt{2}$, so $S = \frac{1}{2} \cdot 3\sqrt{2} \cdot 4\sqrt{2} = 12$. **Answer: C**

Practice 12.

#5 Equilateral

Complete the picture: extend to the equilateral triangle of side 4 whose two other vertices are the arc centers. Its area is $\frac{\sqrt{3}}{4} \cdot 16 = 4\sqrt{3}$; subtract the two 60° sectors: $2 \cdot \frac{1}{6}\pi \cdot 4 = \frac{4\pi}{3}$. **Answer: B**

Practice 13.

#9 Angles & congruence

Each new n-gon adds n sides but glues away 2: $3 + (4-2) + (5-2) + (6-2) + (7-2) + (8-2) = 23$. **Answer: B**

Practice 14.

#5 Equilateral

Equal perimeters → the hexagon's side is *half* the triangle's side s. A regular hexagon splits into 6 equilateral triangles of side $\frac{s}{2}$, and each of those has $\frac{1}{4}$ the area of a side-s equilateral triangle, i.e. area 1. So the hexagon's area is $6 \cdot \frac{4}{4} \cdot \frac{1}{4} = 6 \cdot 1 = 6$. **Answer: C**

Practice 15.

🧰 #1 Triples

Hexagon: $6 \cdot 5 = 30 > 24$ ✗. Square: area 36 → side 6 → perimeter 24 ✓. Right triangle with legs $6, 8$: that's the doubled 3-4-5, so the hypotenuse is 10 and the perimeter is $6 + 8 + 10 = 24$ ✓. **Answer: D**

Practice 16.

🧰 #7 Build a right triangle

Follow one side of the shaded square: it goes 3 units across and 1 unit down (the half-unit brick shifts mean grid vertices sit every half unit along each horizontal line, so this lands on a vertex). By the Pythagorean theorem, $\text{side}^2 = 3^2 + 1^2 = 10$ — and side^2 *is* the square's area. **Answer: A**

Practice 17.

🧰 #3 45-45-90

Each mountain is a 45-45-90 "roof": height 8 → base 16 → area 64; height 12 → base 24 → area 144. Together 208. The overlap (counted twice) is $208 - 183 = 25$. The overlap is itself an upside-down 45-45-90 triangle with apex height h, so its area is h^2. Thus $h^2 = 25$, $h = 5$. **Answer: B**

Practice C1.

🧰 #4 30-60-90 #9 Angles & congruence

$\angle AMD = \angle CDM$ (alternate angles), so $\angle CDM = \angle CMD$, giving $CM = CD = 6$. In right $\triangle CBM$: $CB = 3 = \frac{1}{2}CM$ — a 30-60-90! So $\angle CMB = 30°$ and $\angle AMD = (180 - 30)/2 = 75°$.
Answer: E

Practice C2.

🧰 #4 30-60-90 #7 Build a right triangle #8 Radicals

$AC = BC = 5$, $AB = 5\sqrt{2}$. Since $\triangle CDE$ and $\triangle CAB$ share the altitude from C, $\frac{S_{\triangle CDE}}{12.5} = \frac{DE}{AB}$. Drop $EF \perp BC$; with $\angle ECB = 30°$, $\angle B = 45°$: $EF = x$, $CF = x\sqrt{3}$, $FB = x$, so $x(\sqrt{3} + 1) = 5$ and $BE = x\sqrt{2}$. Solving: $x = \frac{5}{\sqrt{3}+1} = \frac{5(\sqrt{3}-1)}{2}$, so $BE = AD = \frac{5\sqrt{6}-5\sqrt{2}}{2}$ and $DE = 5\sqrt{2} - (5\sqrt{6} - 5\sqrt{2}) = 10\sqrt{2} - 5\sqrt{6}$. Then $S_{\triangle CDE} = 12.5 \cdot \frac{DE}{5\sqrt{2}} = \frac{50-25\sqrt{3}}{2}$. **Answer: D**