

Logic & Data Analysis · Lecture 09 · logic-data-analysis

 **Course materials — Lecture 09 (95 min):** lecture video · handout PDF (Lecture 9) · board notes (09.jpg) · homework answer key (Chapter 9) — all in the `amc8/video` folder. ·  [Printable PDF](#) (answers & solutions at the back)

Key facts

Logic: a statement "if P then Q " is *equivalent* to its contrapositive "if not Q then not P " — and **not** equivalent to its converse or inverse. Two statements that cannot both be true form a *contradictory pair*: exactly one of them is true.

Data: for a sorted list — **mean** = $\frac{\text{sum}}{\text{count}}$; **median** = middle value (average of the two middle values when the count is even); **mode** = most frequent value; **range** = $\text{max} - \text{min}$; **mid-range** = $\frac{\text{max} + \text{min}}{2}$.

Sets (Venn): $|A \cup B| = |A| + |B| - |A \cap B|$ — "or" means the union, "both" means the intersection.

Instructor's toolbox (from the lecture video & board notes)

1 · The three weapons. The instructor opens the lecture with them: **hunt for a contradiction**, **eliminate** (test the choices / cross out impossible cases), **make a table**. Every AMC 8 logic puzzle falls to one of these three — when stuck, switch weapons.

2 · Contrapositive — the only safe flip. The board example: "If I am in Pudong, then I am in Shanghai." Equivalent: "If I am *not in Shanghai*, then I am *not in Pudong*" (contrapositive ✓). NOT equivalent: "If I am in Shanghai, then I am in Pudong" (converse ✗) and "If I am not in Pudong, then I am not in Shanghai" (inverse ✗). So to *verify* "if star, then triangle", you must also check every card that could break the contrapositive: "if no triangle, then no star" (Example 1).

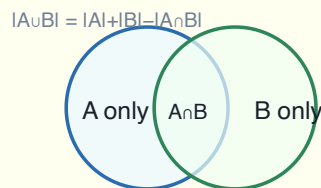
3 · Spend the truth budget on a contradictory (or consistent) pair. When "exactly k of these statements are true", first find two statements that **contradict** each other — exactly one of that pair is true, which fixes the truth of all the rest (Examples 2–3). Two **consistent** statements (one forces the other) live or die *together* — if only one statement in total is true, a consistent pair must both be false (Practice 1–2).

4 · Ranking puzzles: turn every remark into an inequality. Whoever was *shown* a score is secretly comparing with it. Board shorthand for Practice 3: Cassie saw Hannah's score and says "I'm not the lowest" $\Rightarrow H < C$; Bridget: "I'm not the highest" $\Rightarrow H > B$. Chain: $C > H > B$ — done, no cases needed.

5 · Median mechanics: sort first, then count positions. With n values the median sits at position $\frac{n+1}{2}$; for even n it is the *average of the two middle values* — 20 scores $\rightarrow$ positions 10 and 11 (Example 5 turns entirely on those two positions). Know the whole stat family: range = $\max - \min$, mode = most frequent, mid-range = $\frac{\max + \min}{2}$ — Practice 5's trap is that a new minimum moves the range and mid-range but may leave the median and mode untouched.

6 · Squeeze the median: push everything else to its extreme. To make a median as large (small) as possible, make every value on the other side as small (large) as the rules allow — the total is fixed, so whatever you save goes to the middle. Beverage Barn (Practice 8): 49 customers buy just 1 can each, positions 52–100 are capped at the median's own ceiling, and the leftover $252 - 49 = 203$ cans among 51 customers give $203 \div 51 = 3$ r 50 — so positions 50 and 51 get 3 and 4.

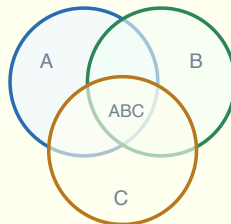
7 · Two-circle Venn: fill from the middle out. $|A \cup B| = |A| + |B| - |A \cap B|$ — adding $|A|$ and $|B|$ counts the overlap twice, so subtract it once. Word cues: "or / at least one" $\rightarrow$ union, "both" $\rightarrow$ intersection, "neither" $\rightarrow$ outside both circles (subtract from the total first — Practice 11). Always write the *overlap* number into the diagram first, then the "only" parts.



8 · Three circles: subtract the doubles once, add the triple back.

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

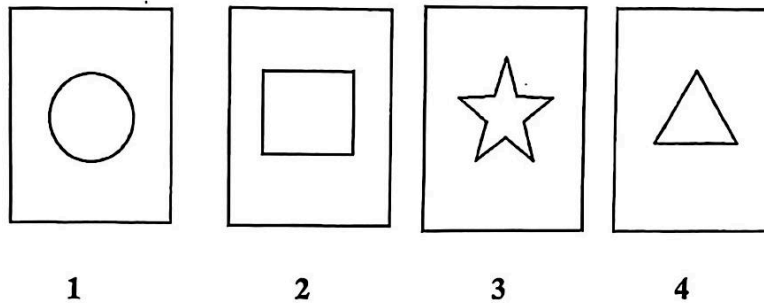
The instructor's memo: elements in exactly two sets got counted twice — subtract once; elements in all three got counted 3 times then subtracted 3 times — add them back. For "every 3 / 4 / 5 days" problems, count multiples with the floor: $\lfloor \frac{365}{3} \rfloor = 121$, $\lfloor \frac{365}{12} \rfloor = 30$ (both 3 and 4 = multiples of 12), $\lfloor \frac{365}{60} \rfloor = 6$ (Practice 13).



Worked examples

course problem · contrapositive

Example 1. Each card has either a circle or a star on one side, and either a triangle or a square on the other side. To verify the statement "every card with a star on it also has a triangle on it," which numbered card(s) must be turned over?



#2 Contrapositive

Key idea: check the statement *and* its contrapositive ("every card without a triangle has no star") — nothing else can break it. Card 3 (star): its back must be a triangle — flip it. Card 2 (square = no triangle): its back must not be a star — flip it. Card 1 (circle): a circle card makes no claim — the statement only restricts *star* cards. Card 4 (triangle): a triangle back is fine no matter what's on the front (the statement is not "triangle $\Rightarrow$ star"). **Answer: cards 2 and 3**

2017 AMC 8 #8 · contradiction

Example 2. Malcolm wants to visit Isabella after school today and knows the street where she lives but doesn't know her house number. She tells him, "My house number has two digits, and exactly three of the following four statements about it are true." (1) It is prime. (2) It is even. (3) It is divisible by 7. (4) One of its digits is 9. This information allows Malcolm to determine Isabella's house number. What is its units digit?

(A) 4 (B) 6 (C) 7 (D) 8 (E) 9



#3 Contradictory pair

Key idea: a two-digit number can't be both prime and even — (1) and (2) contradict, so the single false statement is one of them. But (1) also contradicts (3) (a two-digit multiple of 7 isn't prime), so (1) is the false one, and (2), (3), (4) are all true. Two-digit even multiples of 7: 14, 28, 42, 56, 70, 84, 98. With a digit 9: only 98. Units digit 8. **Answer: D**

Example 3. There are three boxes of different colors: red, yellow, and blue. An apple is in one of the three boxes. Exactly one of the following statements is true, and the others are false: I. The apple is in the red box. II. The apple is not in the yellow box. III. The apple is not in the red box. Which box is the apple in?



#3 Contradictory pair

Key idea: I and III contradict each other, so the one true statement lives in that pair — which means II is **false**. "Not in the yellow box" being false puts the apple *in the yellow box*. (Check: then III is true, I is false — exactly one true ✓.) **Answer: the yellow box**

Example 4. A scientist walking through a forest recorded as integers the heights of 5 trees standing in a row. She observed that each tree was either twice as tall or half as tall as the one to its right. Unfortunately some of her data was lost when rain fell on her notebook. Her notes are shown below, with blanks indicating the missing numbers. Based on her observations, the scientist was able to reconstruct the lost data. What was the average height of the trees, in meters?

Tree 1	___ meters
Tree 2	11 meters
Tree 3	___ meters
Tree 4	___ meters
Tree 5	___ meters
Average height	___ .2 meters

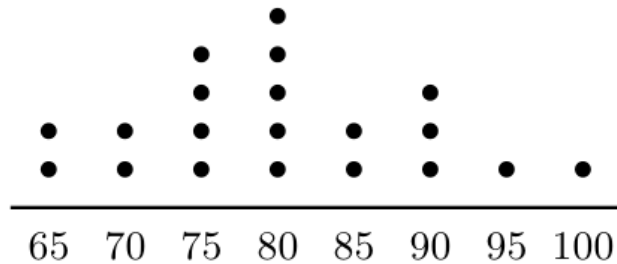
(A) 22.2 (B) 24.2 (C) 33.2 (D) 35.2 (E) 37.2



#1 Eliminate cases

Key idea: heights are integers, so no neighbor of 11 can be 5.5 — trees 1 and 3 are both 22. The average ends in .2, so the total ends in 1 or 6 (total = $5 \times \text{average}$). Tree 4 is 44 or 11: if 11, tree 5 is 22 (integer), total $22 + 11 + 22 + 11 + 22 = 88$ ✗. So tree 4 = 44; tree 5 is 88 or 22: total 187 ✗ or 121 ✓ (ends in 1). Average = $121 \div 5 = 24.2$. **Answer: B**

Example 5. Mr. Ramos gave a test to his class of 20 students. The dot plot below shows the distribution of test scores. Later Mr. Ramos discovered that there was a scoring error on one of the questions. He regraded the tests, awarding some of the students 5 extra points, which increased the median test score to 85. What is the minimum number of students who received extra points? (Note that the median test score equals the average of the 2 scores in the middle if the 20 test scores are arranged in increasing order.)



- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6



#5 Median positions

Key idea: only positions 10 and 11 matter. Before regrading, positions 10–13 all score 80 (median 80). For the new median to be 85, the 10th and 11th scores must both become 85. But if you bump only two of the four 80s, the two untouched 80s slide *down* into positions 10–11 and the median stays 80 — so all four 80s (positions 10, 11, 12, 13) need the +5. **Answer: C**

Practice

Try each one before opening the solution. Figures are from the official exams and course handouts.

1. A car is hidden in one of three boxes, and each box has a sign on it. Exactly one of the three signs is true. Which box is the car in?

Box 1

"The car is in this box."

Box 2

"The car is not in this box."

Box 3

"The car is not in box 1."

2. Each of three marbles A , B , and C is colored one of three colors: one is white, one is red, and one is blue. Exactly one of these statements is true: 1) A is red. 2) B is not blue. 3) C is not red. What color is marble B ?

2013 AMC 8 #19

3. Bridget, Cassie, and Hannah are discussing the results of their last math test. Hannah shows Bridget and Cassie her test, but Bridget and Cassie don't show theirs to anyone. Cassie says, "I didn't get the lowest score in our class," and Bridget adds, "I didn't get the highest score." What is the ranking of the three girls from highest to lowest?

- (A) Hannah, Cassie, Bridget (B) Hannah, Bridget, Cassie (C) Cassie, Bridget, Hannah
(D) Cassie, Hannah, Bridget (E) Bridget, Cassie, Hannah

2001 AMC 8 #20

4. Kaleana shows her test score to Quay, Marty and Shana, but the others keep theirs hidden. Quay thinks, "At least two of us have the same score." Marty thinks, "I didn't get the lowest score." Shana thinks, "I didn't get the highest score." List the scores from lowest to highest for Marty (M), Quay (Q) and Shana (S).

- (A) S, Q, M (B) Q, M, S (C) Q, S, M (D) M, S, Q (E) S, M, Q

2015 AMC 8 #5

5. Billy's basketball team scored the following points over the course of the first 11 games of the season:

42, 47, 53, 53, 58, 58, 58, 61, 64, 65, 73

If his team scores 40 in the 12th game, which of the following statistics will show an increase?

- (A) range (B) median (C) mean (D) mode (E) mid-range

2004 AMC 8 #11

6. The numbers -2 , 4 , 6 , 9 and 12 are rearranged according to these rules: (1) the largest isn't first, but it is in one of the first three places; (2) the smallest isn't last, but it is in one of the last three places; (3) the median isn't first or last. What is the average of the first and last numbers?

- (A) 3.5 (B) 5 (C) 6.5 (D) 7.5 (E) 8

7. The mean, median, and unique mode of the positive integers 3, 4, 5, 6, 6, 7, and x are all equal. What is the value of x ?

- (A) 5 (B) 6 (C) 7 (D) 11 (E) 12

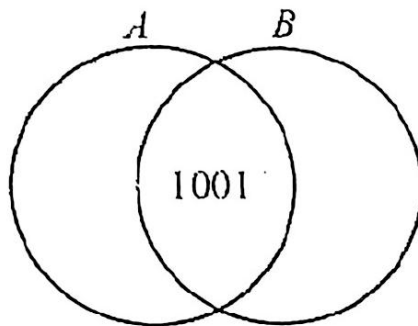
8. One day the Beverage Barn sold 252 cans of soda to 100 customers, and every customer bought at least one can of soda. What is the maximum possible median number of cans of soda bought per customer on that day?

- (A) 2.5 (B) 3.0 (C) 3.5 (D) 4.0 (E) 4.5

9. In a town of 351 adults, every adult owns a car, motorcycle, or both. If 331 adults own cars and 45 adults own motorcycles, how many of the car owners do not own a motorcycle?

- (A) 20 (B) 25 (C) 45 (D) 306 (E) 351

10. Sets A and B , shown in the Venn diagram, have the same number of elements. Their union has 2007 elements and their intersection has 1001 elements. Find the number of elements in A .



- (A) 503 (B) 1006 (C) 1504 (D) 1507 (E) 1510

11. At Euler Middle School, 198 students voted on two issues in a school referendum with the following results: 149 voted in favor of the first issue and 119 voted in favor of the second issue. If there were exactly 29 students who voted against both issues, how many students voted in favor of both issues?

- (A) 49 (B) 70 (C) 79 (D) 99 (E) 149

2019 AMC 8 #15

12. On a beach 50 people are wearing sunglasses and 35 people are wearing caps. Some people are wearing both sunglasses and caps. If one of the people wearing a cap is selected at random, the probability that this person is also wearing sunglasses is $\frac{2}{5}$. If instead, someone wearing sunglasses is selected at random, what is the probability that this person is also wearing a cap?

- (A) $\frac{14}{85}$ (B) $\frac{7}{25}$ (C) $\frac{2}{5}$ (D) $\frac{4}{7}$ (E) $\frac{7}{10}$

2017 AMC 8 #24

13. Mrs. Sanders has three grandchildren, who call her regularly. One calls her every three days, one calls her every four days, and one calls her every five days. All three called her on December 31, 2016. On how many days during the next year did she not receive a phone call from any of her grandchildren?

- (A) 78 (B) 80 (C) 144 (D) 146 (E) 152

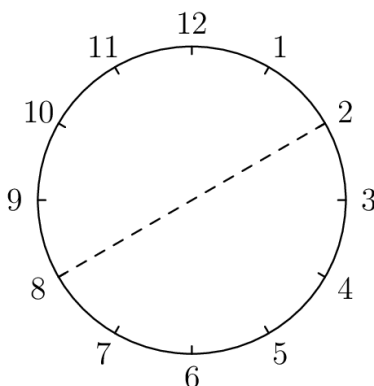


Fresh from the latest exams

The same ideas, exactly as they appeared on the most recent tests.

2025 AMC 8 #9

14. Ningli looks at the 6 pairs of numbers directly across from each other on a clock. She takes the average of each pair of numbers. What is the average of the resulting 6 numbers?



- (A) 5 (B) 6.5 (C) 8 (D) 9.5 (E) 12

2025 AMC 8 #14

15. A number N is inserted into the list 2, 6, 7, 7, 28. The mean is now twice as great as the median. What is N ?

- (A) 7 (B) 14 (C) 20 (D) 28 (E) 34

16. The integers from 1 to 25 are arbitrarily separated into five groups of 5 numbers each. The median of each group is identified. Let M equal the median of the five medians. What is the least possible value of M ?

- (A) 9 (B) 10 (C) 12 (D) 13 (E) 14

★ Challenge (AMC 10 level)

2021 AMC 10A #7 · contrapositive

C1. Tom has a collection of 13 snakes, 4 of which are purple and 5 of which are happy. He observes that: all of his happy snakes can add; none of his purple snakes can subtract; and all of his snakes that can't subtract also can't add. Which of these conclusions can be drawn about Tom's snakes?

- (A) Purple snakes can add. (B) Purple snakes are happy. (C) Snakes that can add are purple.
(D) Happy snakes are not purple. (E) Happy snakes can't subtract.

2010 AMC 10A #15 · truth-tellers & liars

C2. In a magical swamp there are two species of talking amphibians: toads, whose statements are always true, and frogs, whose statements are always false. Four amphibians, Brian, Chris, LeRoy, and Mike, live together in this swamp, and they make the following statements. Brian: "Mike and I are different species." Chris: "LeRoy is a frog." LeRoy: "Chris is a frog." Mike: "Of the four of us, at least two are toads." How many of these amphibians are frogs?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

2018 AMC 10B #14 · mode

C3. A list of 2018 positive integers has a unique mode, which occurs exactly 10 times. What is the least number of distinct values that can occur in the list?

- (A) 202 (B) 223 (C) 224 (D) 225 (E) 234

course problem · three-set Venn

C4. There are 20 students participating in an after-school program offering classes in yoga, bridge, and painting. Each student must take at least one of these three classes, but may take two or all three. There are 10 students taking yoga, 13 taking bridge, and 9 taking painting. There are 9 students taking at least two classes. How many students are taking all three classes?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Sources: prep-course Lecture 09 (video + board notes + 22 worked examples, translated to English) and official AMC 8 / AMC 10 problems from the local bank ([kb/content/banks/](#)) and course handouts. Answers verified against official answer keys where available.

Answer key

Example	1	2	3	4	5											
	cards 2 & 3	D	yellow box	B	C											
Practice	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	box 2	white	D	A	A	C	D	C	D	C	D	B	D	B	E	A
Challenge	C1	C2	C3	C4												
	D	D	D	C												

Solutions to practice problems

Practice 1.



#3 Contradictory pair

Signs 1 and 3 contradict each other, so the single true sign is one of those two — sign 2 must be **false**. A false "the car is not in this box" means the car *is* in box 2. (Check: car in box 2 makes sign 1 false, sign 3 true — exactly one true ✓. Note signs 1 and 2 are also *consistent*: sign 1 true would force sign 2 true, two truths — impossible, so sign 1 is false either way.) **Answer: box 2**

Practice 2.



#3 Consistent pair

Statements 1) and 3) are *consistent*: if A is red then C certainly isn't — 1) true forces 3) true. So 1) and 3) can't split; with only one truth available they must **both be false**, and 2) is the true one. 3) false $\Rightarrow C$ is red. 2) true $\Rightarrow B$ is not blue, so B is white (and A is blue). **Answer: white**

Practice 3.



#4 Remarks $\rightarrow$ inequalities

Both girls are comparing themselves with the one score they saw — Hannah's. Cassie, "not the lowest": she beats someone she knows about $\Rightarrow C > H$. Bridget, "not the highest": $\Rightarrow B < H$. Chain: $C > H > B$. **Answer: D**

Practice 4.



#4 Remarks → inequalities

Everyone is comparing with Kaleana's score K , the only one they saw. Quay's "at least two of us are equal" can only mean $Q = K$. Marty's "not the lowest" $\Rightarrow M > K$. Shana's "not the highest" $\Rightarrow S < K$. So $S < Q = K < M$: from lowest to highest, S, Q, M. **Answer: A**

Practice 5.



#5 Stat family

40 is a new *minimum*. Range = max – min: the min drops, so the range **grows** ✓. Median: 12 values now, average of the 6th and 7th = $\frac{58+58}{2} = 58$ — unchanged. Mean: a below-average value pulls it down. Mode: still 58. Mid-range = $\frac{73+40}{2}$ — smaller. **Answer: A**

Practice 6.



#1 Eliminate

#5 Median

Rule (1): 12 is in place 2 or 3. Rule (2): -2 is in place 3 or 4. Rule (3): the median 6 is in place 2, 3, or 4. So places 1 and 5 can only hold 4 and 9 (in some order) — average $\frac{4+9}{2} = 6.5$. **Answer: C**

Practice 7.



#5 Stat family

x can't be 3, 4, 5 or 7 — that would create a second mode. So the unique mode is 6, which must also be the mean: $\frac{3+4+5+6+6+7+x}{7} = 6 \Rightarrow x = 42 - 31 = 11$. (Sorted with $x = 11$ the median — the 4th value — is indeed 6 ✓.) **Answer: D**

Practice 8.



#6 Squeeze the median

#5 Median positions

Sort the 100 purchases; the median is the average of positions 50 and 51. Starve the left side: customers 1–49 buy just 1 can each, leaving $252 - 49 = 203$ cans for the 51 customers in positions 50–100. Could the median be 4? Then positions 50–100 each need ≥ 4 cans: $51 \cdot 4 = 204 > 203$ ✗ — one can short. So aim for 3.5: positions 52–100 take 4 cans each ($49 \cdot 4 = 196$), leaving $203 - 196 = 7$ for positions 50 and 51: they get 3 and 4 ✓ (sorted order holds). Median = $\frac{3+4}{2} = 3.5$. **Answer: C**

Practice 9.



#7 Two-circle Venn

$|A \cup B| = 351$, so $351 = 331 + 45 - |A \cap B| \Rightarrow |A \cap B| = 25$ own both. Car owners without a motorcycle: $331 - 25 = 306$. **Answer: D**

Practice 10.



#7 Two-circle Venn

$|A \cup B| = |A| + |B| - |A \cap B|$ with $|A| = |B|$: $2007 = 2|A| - 1001 \Rightarrow |A| = 1504$. **Answer: C**

Practice 11.



#7 Two-circle Venn

"Against both" lives *outside* both circles — remove it first: $|A \cup B| = 198 - 29 = 169$. Then $169 = 149 + 119 - |A \cap B| \Rightarrow |A \cap B| = 99$. **Answer: D**

Practice 12.



#7 Two-circle Venn

Both probabilities share the same overlap. From the cap side: both $= 35 \cdot \frac{2}{5} = 14$. From the sunglasses side: $P = \frac{14}{50} = \frac{7}{25}$. **Answer: B**

Practice 13.



#8 Three-circle Venn

Number the days 1–365; the calls land on multiples of 3, 4, 5. Singles: $\lfloor \frac{365}{3} \rfloor = 121$, $\lfloor \frac{365}{4} \rfloor = 91$, $\lfloor \frac{365}{5} \rfloor = 73$. Pairs (lcm): $\lfloor \frac{365}{12} \rfloor = 30$, $\lfloor \frac{365}{15} \rfloor = 24$, $\lfloor \frac{365}{20} \rfloor = 18$. Triple: $\lfloor \frac{365}{60} \rfloor = 6$. Days with a call: $121 + 91 + 73 - (30 + 24 + 18) + 6 = 219$. Days without: $365 - 219 = 146$. **Answer: D**

Practice 14.



#5 Mean

An average of averages over *equal-size* groups is just the average of everything: $\frac{1 + 2 + \dots + 12}{12} = \frac{78}{12} = 6.5$. (Slow way: the pairs (12, 6), (1, 7), ..., (5, 11) average to 9, 4, 5, 6, 7, 8 — mean $\frac{39}{6} = 6.5$.) **Answer: B**

Practice 15.



#5 Median positions

#1 Eliminate

Six values: the median is the average of the 3rd and 4th. If $N \geq 7$: sorted list $2, 6, 7, 7, \dots$ — median = $\frac{7+7}{2} = 7$, so the mean is 14 and the sum is 84: $N = 84 - 50 = 34$ ✓ (consistent with $N \geq 7$). If $N \leq 6$: median = $\frac{6+7}{2} = 6.5$, mean 13, sum 78, $N = 28$ — contradicts $N \leq 6$ ✗. **Answer: E**

Practice 16.



#6 Squeeze the median

M is the 3rd-smallest of the five group medians, so make three medians tiny and let the other two blow up. Construction: groups starting $\{1, 2, 3, *, *\}$, $\{4, 5, 6, *, *\}$, $\{7, 8, 9, *, *\}$ (big numbers fill the *) have medians 3, 6, 9 — so $M = 9$. Why no lower: each of the three smallest medians needs 2 group members below it, so at least $3 \cdot 3 = 9$ distinct integers are $\leq$ the third median — it is at least 9. **Answer: A**

Practice C1.



#2 Contrapositive

Chain the facts: purple $\Rightarrow$ can't subtract $\Rightarrow$ can't add. The contrapositive of "happy $\Rightarrow$ can add" is "can't add $\Rightarrow$ not happy" — so purple $\Rightarrow$ not happy, and (contrapositive again) happy $\Rightarrow$ not purple ✓ (D). The others fail: (A) purple snakes *can't* add; (B) purple snakes aren't happy; (C) reverses an implication (converse!); (E) happy $\Rightarrow$ can add $\Rightarrow$ *can* subtract (contrapositive of fact 3). **Answer: D**

Practice C2.



#3 Contradictory pair

#1 Case check

Chris and LeRoy accuse each other — if both were toads, both statements would be true, making both frogs (contradiction); both frogs is symmetric. So **exactly one** of Chris/LeRoy is a toad. Now test Brian: if Brian is a toad, Mike is a frog, giving 2 toads (Brian + one of C/L) — but then Mike's "at least two are toads" would be *true* from a frog ✗. So Brian is a frog; his statement is false, so Mike is also a frog. Now only one toad exists (among C/L), so Mike's statement is false ✓ consistent. Frogs: Brian, Mike, and one of Chris/LeRoy — three. **Answer: D**

Practice C3.



#6 Push to the extreme

#5 Mode

To use as *few* distinct values as possible, repeat every other value as often as allowed — at most 9 times each (the mode of 10 must stay unique). Non-mode occurrences: $2018 - 10 = 2008 = 9 \cdot 223 + 1$ — so 223 values used 9 times and 1 more value used once. Total distinct: $223 + 1 + 1 = 225$. **Answer: D**

Practice C4.



#8 Three-circle Venn

Let $P = |A \cap B| + |A \cap C| + |B \cap C|$ and $T = |A \cap B \cap C|$. Inclusion–exclusion: $20 = 10 + 13 + 9 - P + T = 32 - P + T$. "At least two classes" counts each double once and each triple once — but P counts triples three times, so $P - 2T = 9$. Substitute $P = 9 + 2T$: $20 = 32 - (9 + 2T) + T = 23 - T \Rightarrow T = 3$. **Answer: C**